IOP Conf. Series: Journal of Physics: Conf. Series **944** (2018) 012060 doi:10.1088/1742-6596/944/1/012060

Rationale choosing interval of a piecewise-constant approximation of input rate of non-stationary queue system

Ivan A Korelin[1] and Sergey V Porshnev[1]
[1] The Institute of Radioelectronics and Information Technologies, Ural Federal University, Mira st. 32, Yekaterinburg, Russia 620002

Abstract. The paper demonstrates the possibility of calculating the characteristics of the flow of visitors to objects carrying out mass events passing through checkpoints. The mathematical model is based on the non-stationary queuing system (NQS) where dependence of requests input rate from time is described by the function. This function was chosen in such way that its properties were similar to the real dependencies of speed of visitors arrival on football matches to the stadium. A piecewise-constant approximation of the function is used when statistical modeling of NQS performing. Authors calculated the dependencies of the queue length and waiting time for visitors to service (time in queue) on time for different laws. Time required to service the entire queue and the number of visitors entering the stadium at the beginning of the match were calculated too. We found the dependence for macroscopic quantitative characteristics of NQS from the number of averaging sections of the input rate.

1. Introduction

Conclusions and regularities obtained as a result of the development of queuing theory (QT) have found application in a variety of fields and sciences: astronomy, agronomy, geology, hydrology, teletraffic theory, chemistry, economics, ecology, physics and statistics [1]. However, most of the analytical results in QT are obtained for stationary streams of requests (first of all, a Poisson flow) [1, 2]. Such streams of requests are extremely rare in practice, because in reality, the assumption of their stationarity is not met, in most cases. For example this is the case of access control systems that provide access to mass events [4], passenger control devices at airports and railway stations, etc. The study of the features of their operation is of undoubted interest from a practical point of view, since the results obtained can be used as a scientific justification for the design decisions made at the modernization or design stage of non-stationary queuing systems (NQS).

In the conducted research, authors used an approach of the quantitative characteristics of the NQS study, which was previously demonstrated by the authors in [3]. In this approach a piecewise constant approximation of the function $\lambda = \lambda(t)$ is used. This is mean that at each interval of the piecewise constant approximation of the function $\lambda = \lambda(t)$, a stationary stream of requests with rate λ_k arrives at the input of the QS. The lengths of the queue at the entrance to the QS are calculated by accounting the applications received at the input of the QS at the previous intervals of approximation and not served till the time τ_k. It is clear that one of the main questions arising in the practical use of this approach is to chose of the length of the interval of the piecewise constant approximation of the function $\lambda = \lambda(t)$.

This article presents the results of statistical modeling of the NQS in accordance with the approach described above and provides recommendations on the choice of the duration of the interval of piecewise-constant approximation, depending on the parameters of the function $\lambda = \lambda(t)$.

2. Mathematical model of the non-stationary queue system

The block diagram of the model of a non-stationary single-channel SMO with an unlimited queue is presented in Figure 1.

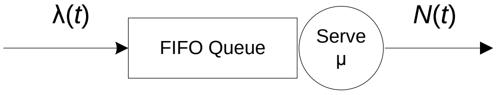

Figure 1. The scheme model of non-stationary QS

As you can see from figure 1, in this model QS processes a flow of requests with the rate $\lambda = \lambda(t)$ varying with time. The main characteristic of the input stream is the instantaneous flux density $\lambda(t)$.This characteristic means the limit of the ratio of the average number of events for a time (t, t+Δt), to the length of this section, provided that the length of the segment tends to zero:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{M(t+\Delta t) + M(t)}{\Delta t} = \frac{dM(t)}{dt} \tag{1}$$

The M(t) is expected value of the number of events on the site (0, t).

The FIFO policy (the first one in is the first one out) used to serve the queue of visitors. The service speed of incoming requests is determined by the service rate $\bar{\mu}$, which is a random variable with a probability density

$$p\{\xi\} = \begin{cases} 0, \text{ then } \xi < 1, \\ \dfrac{2}{9(M[\xi]-1)}(\xi - 1), \text{ then } 1 \le \xi < M[\xi], \\ \dfrac{2}{9(M[\xi]-10)}(\xi - 10), \text{ then } M[\xi] < \xi \le 10, \\ 0, \text{ then } \xi > 10, \end{cases} \tag{2}$$

then $\xi \in [1,10]$. In this study, a distribution law is chosen such that $M[\xi] = 4$ and $\bar{\mu} = 15$.

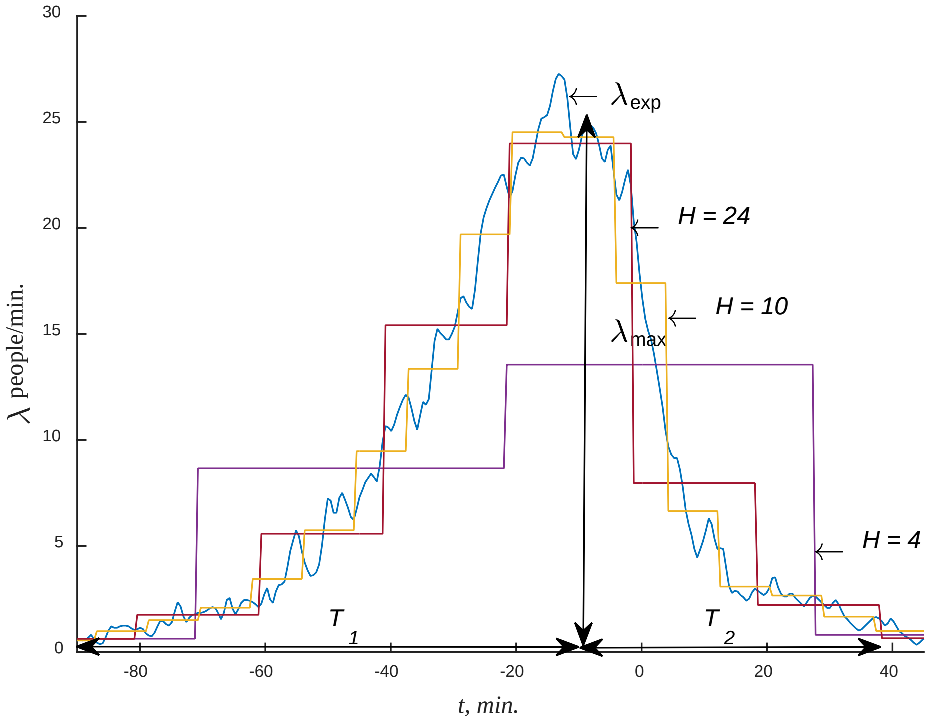

Figure 2. The dependencies $\lambda(t)$ for non-stationary QS (beginning of match t = 0).

On figure 2 there is a typical dependence $\lambda_{exp}(t)$, wich was received during the football match between the football clubs "Krylia Sovetov" and "Dynamo" at the stadium "Metallurg" in Samara 05.05.2013 [4].

Figure 2 shows that access to the stadium "Metallurg" was opened 1.5 hours before the start of the football match. After the opening of the turnstiles during the time $T_1 \approx 70$ min. input rate of requests increased from 0 people/min. to $\lambda_{max}=28$ people/min. Approximately 15 minutes before the match, the intensity decreased from $\lambda_{max}=28$ people/min. to 0 people/min. during the time interval $T_2 \approx 50$ min. Thus mean that total number N of visitors entered through one access control device (turnstile) at the stadium "Metallurg"

$$N = \int_{-80}^{40} \lambda(t)\,dt \tag{3}$$

is 1400 people.

The characteristics of the NQS for the family of functions $\lambda = \lambda(t)$ were studied. These functions on time interval T_1 [–80; –10] minutes are Increased by a quadratic law from 0 to λ_{max}, afterwards on time interval T_2 [–10;40] minutes quadratically decreasing from λ_{max} to 0. Parameter λ_{max} varied from 18 people/min. from 32,4 people/min. with step 0,8, T_1 and T_2 remained unchanged, and the parameters of parabolas were chosen in such a way that the total number of entered N for each λ_{max} would be 1400 people. (Figure 3).

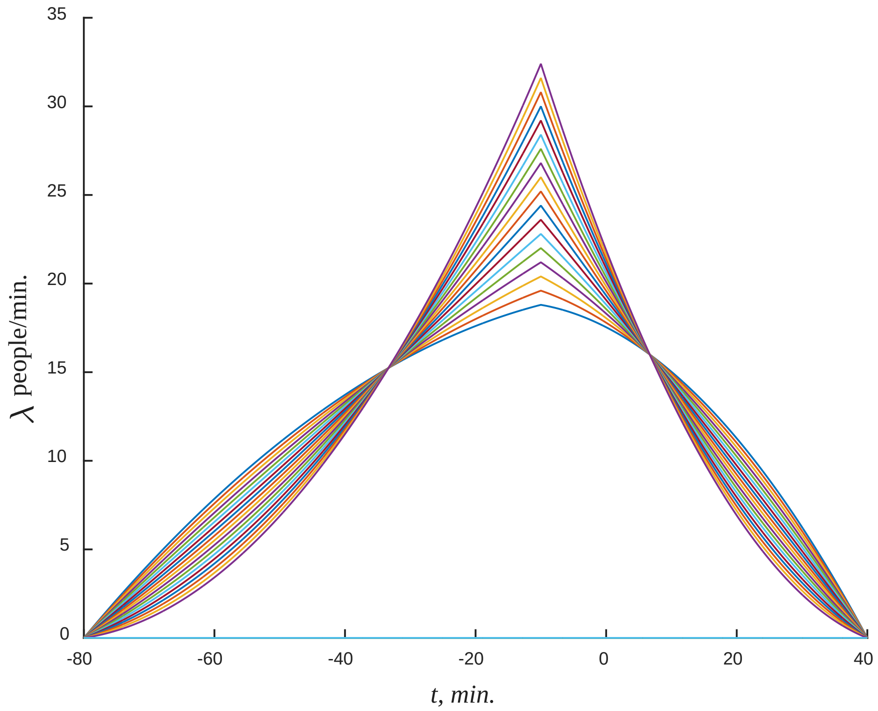

Figure 3. The dependencies of $\lambda(t)$ on the non-stationary QS (the beginning of the match $t = 0$)

In accordance with the chosen approach in carried out computational experiments a piecewise constant approximation of the functions $\lambda = \lambda(t)$ on interval $[T_1, T_2]$ was used. Number of intervals was varied in set $H \in \{2, 4, 8, 16, 32, 64, 100, 154, 205, 308, 616, 1232\}$.

3. Method of computer experiments

The flowchart of the algorithm used in performing the statistical simulation described in detail in [3]. Requests are generating at each interval of the piecewise constant approximation during time interval. The request arrival times t_A have the exponential distribution law with the rate equal to the average value of the input rate of requests on this interval. The service time interval was generated according to formula (2). Further the time for setting up the service was calculated by alternately viewing all the requests that stood on this interval of the service queue:

$$t_{E_i}=\begin{cases} t_{A_i}, \text{then } t_{A_i}>t_{E_{i-1}}+\tau_{S_{i-1}}, \\ t_{A_i}+(t_{E_{i-1}}+\tau_{S_{i-1}}-t_{A_i}), \text{then } t_{A_i}<t_{E_{i-1}}+\tau_{S_{i-1}}. \end{cases} \quad (4)$$

Note that for the first request $t_{E_1}=t_{A_1}$ it is obvious.

The analysis of the characteristics used to describe the features of the operation of NQS [3] showed that for their quantitative description it is possible, for example, to use the dependence of the length of the queue of visitors (in terms of QS - queue length of requests) from time:

$$L=L\left(\lambda_{max},t_k\right)=\left|Q\right|, \text{ then } Q=\{q_n:(t_{A_n}<t_k)\cap(t_{E_n}>t_k)\}, \quad (5)$$

Average waiting time in queue for the visitor (in terms of QS - request) in the queue from time is:

$$\tau_w=\tau_w\left(\lambda_{max},t_k\right)=\frac{\sum_{i=1}^{|Q|}(t_{E_i}-t_{A_i})}{|Q|}, \text{ then } Q=\{q_n:(t_{E_n}\le t_k)\cap(t_{E_n}>t_{k-1})\}, \quad (6)$$

then $t_k=T_1+\frac{T_2-T_1}{K}(k-1)$, $k=\overline{1,K}$, K is the number of intervals of piecewise-constant approximation $\lambda(t)$. Also the number of visitors entering at the beginning of the match:

$$N_0=N_0\left(\lambda_{max}\right)=\left|Q\right|, \text{ then } Q=\{q_n:t_{E_n}<t=0\}, \quad (7)$$

And the time required to service all incoming visitors:

$$T_{All}=T_{All}\left(\lambda_{max}\right)=\{t:N(t)\ge 0.97\cdot N_{\max}\}, \text{ then } N_{\max}=\max(N(t)). \quad (8)$$

Since the Monte Carlo method was used in the simulation, the values of the characteristic function of the non-stationary SMO were taken to be their mean values in the ensemble of independent realizations:

$$\overline{\Phi}\left(\lambda_{max},t_k\right)=\frac{1}{m}\sum_{j=1}^{m}\Phi_j\left(\lambda_{max},t_k\right), \quad (9)$$

then *m* is the number of independent tests in the Monte Carlo method, Φ is element of set $\{L,\tau_w\}$,

$$\overline{\Psi}\left(\lambda_{max}\right)=\frac{1}{m}\sum_{j=1}^{m}\Psi_j\left(\lambda_{max}\right), \quad (10)$$

then Ψ is element of set $\{N_0,T_{All}\}$.

4. Analysis of experimental results

Let us consider the dependences $\overline{L}\left(\lambda_{max},t_k\right)$ for different values λ_{max}, H = 1232 presented in Fig. 4. It is seen from figure that these dependencies can be characterized by two parameters: the maximum value of the function $\overline{L}_{\max}=\max\left(\overline{L}\left(\lambda_{max},t_k\right)\right)$ and the abscissa value of the function $\overline{L}\left(\lambda_{max},t_k\right)$ in which it reaches its maximum value $\overline{\tau}_{\max}=\arg\max_{t_k}\left(\overline{L}\left(\lambda_{max},t_k\right)\right)$. These parameters are some functions that depend on the λ_{max}.

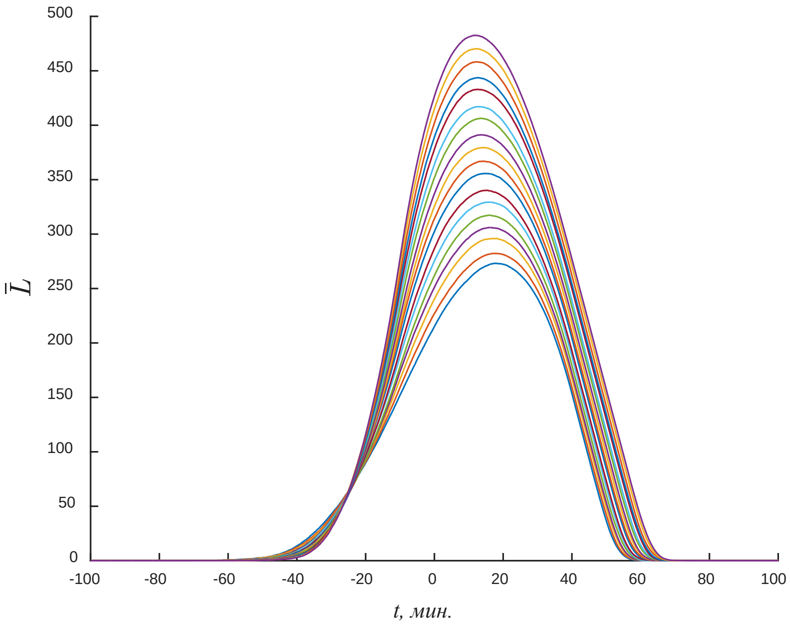

Figure 4. The dependencies $\bar{L}(\lambda_{max}, t)$ (the beginning of the match t = 0) for different values λ_{max}

In this connection, the dependencies of the selected parameters on the number of intervals of the piecewise constant approximation H was calculated. Figure 5 and figure 6, respectively shows dependencies $\bar{L}_{\max} = f(\lambda_{\max}, H)$ and $\bar{\tau}_{\max} = f(\lambda_{\max}, H)$. In addition, there are dependencies $N_0 = f(\lambda_{\max}, H)$ and $T_{All} = f(\lambda_{\max}, H)$ on figure 7 and figure 8.

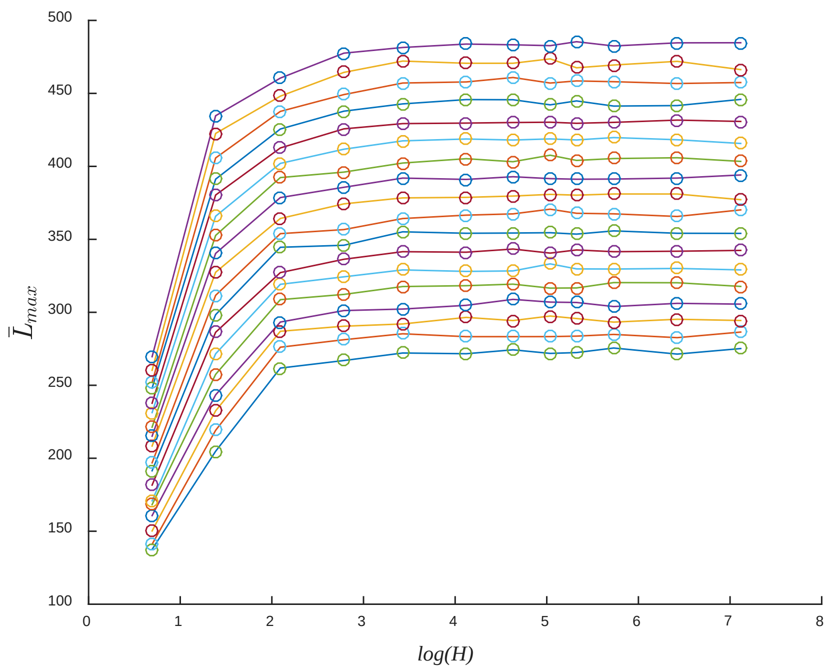

Figure 5. The dependencies $\bar{L}_{\max} = f(\lambda_{\max}, H)$ for the above parameter values λ_{max}

IOP Conf. Series: Journal of Physics: Conf. Series **944** (2018) 012060 doi:10.1088/1742-6596/944/1/012060

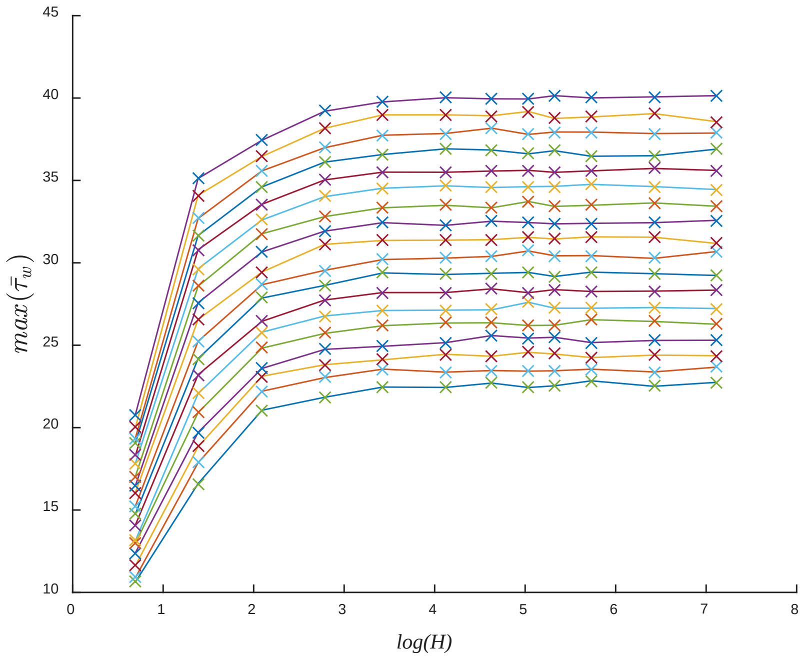

Figure 6. The dependencies $\max(\bar{\tau}_w) = f(\lambda_{\max}, H)$ for the above parameter values λ_{max}

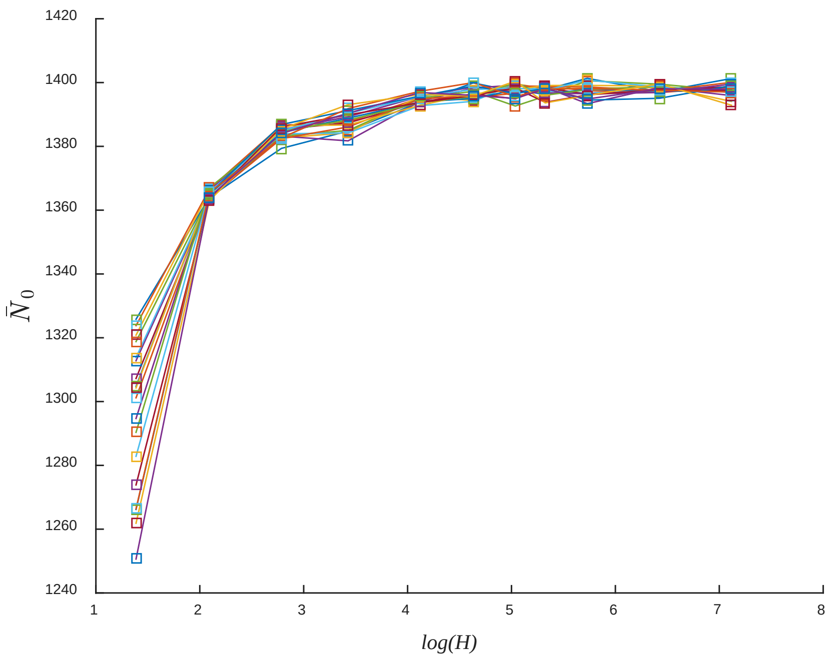

Figure 7. The dependencies $N_0 = f(\lambda_{\max}, H)$ for the above parameter values λ_{max}

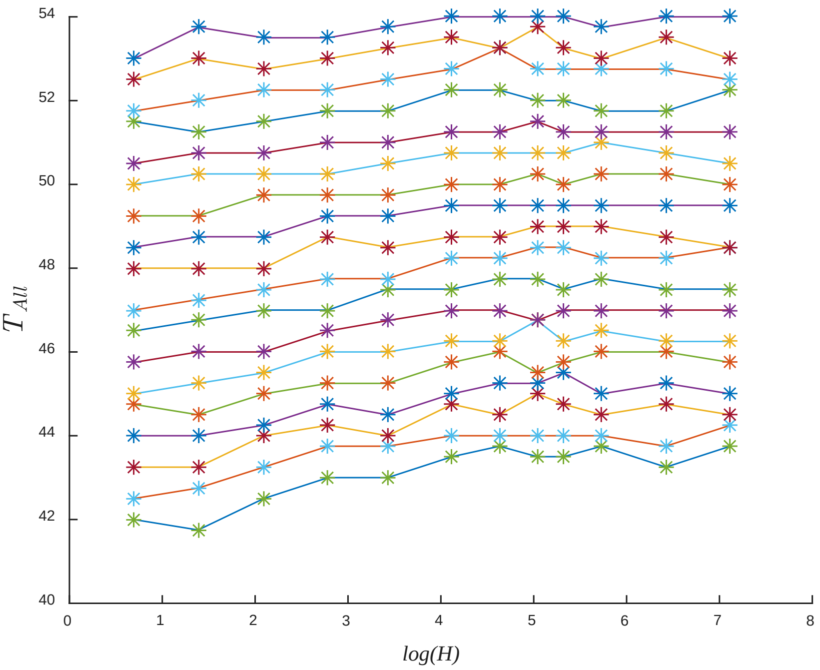

Figure 8. The dependencies $T_{All} = f(\lambda_{\max}, H)$ for the above parameter values λ_{max}

It can be seen from Figures 5-8 that the dependencies $\bar{L}_{\max} = f(\lambda_{\max}, H)$, $\max(\bar{\tau}_w) = f(\lambda_{\max}, H)$, $N_0 = f(\lambda_{\max}, H)$, $T_{All} = f(\lambda_{\max}, H)$ for each of the above values λ_{max} turn out to be similar to each other for values of H greater than 100 (the corresponding length of the interval of the piecewise linear interpolation of the function $\lambda = \lambda(t)$ - 7.5 minutes). The value of these functions practically does not change with even more increasing H. This is mean, when simulating the studied NQS, it is sufficient to use intervals of a piecewise-constant approximation H of less than 7.5 minutes. This ensures that estimates $\bar{L}_{\max}, \max(\bar{\tau}_w), N_0, T_{all}$ independent of H.

At the same time, it is clear that each of these characteristics, being calculated as the mean value over an ensemble of independent realizations, is some random variable. In this connection, the further direction of research will be the study of the influence of the size of the window of piecewise linear approximation on the laws of distribution of random variables.

5. Conclusion

In this paper estimated the number of intervals and, correspondingly, the duration of intervals of piecewise-linear approximation of the function $\lambda = \lambda(t)$, describing the input rate of requests at input of NQS from time. Also counted value number of intervals providing stable estimates of the maximum length of the queue, the maximum waiting time in the queue, the number of visitors entered by the time the function $\lambda = \lambda(t)$, reaches the maximum value, the time to serve all visitors.

In further publications it is need to be valuate effect the size of window piecewise-constant approximation to the laws of the distribution of random variables $\bar{L}_{\max}, \bar{\tau}_{\max}, \bar{N}_0, \bar{T}_{all}$.

6. References

[1] Gnedenko B V and Kovalenko I N 1966 *Vvedenie v teoriju massovogo obsluzhivanija* [Introduction to queuing theory] Moscow: Nauka Publ. p 432

[2] Kendall D G 1953 Stochastic process occurring in the theory of queues and their analysis by the method of the imbedded markov chain *The Annals of Mathematical Statistics*. **24** (3): p 338 doi:10.1214/aoms/1177728975. JSTOR 2236285

[3] Porshnev S and Korelin I, 2017 Non-Stationary Single-Channel Queuing System Features Research in Context of Number of Served Queries *ITM Web Conf*. **10** 03006 p 4 doi: 10.1051/itmconf/20171003006

[4] Porshnev S V, Jakob D A 2014 *Issledovanie osobennostej funkcionirovanija informacionnyh kontrol'no-propusknyh sistem ob'ektov provedenija massovyh meroprijatij* [The study of the functioning of information systems checkpoints objects of mass events], Ekaterinburg: Institut jekonomiki UrO RAN Publ. p 216.